

## **Appendix B. Uncertainty Sample Calculations**

# Calibration Uncertainties for the Fuel Cell Testing Facility

## *Calibration uncertainty calculation for voltage measurement.*

### Model Equations

$$V_i = m \times V_{FP} + c \quad \text{Equation 1}$$

$$V_i = V_{dmm} \quad \text{Equation 2}$$

### Sensitivity determination

$$\frac{\partial V_i}{\partial V_{dmm}} = 1 \quad \text{Equation 3}$$

$$\frac{\partial V_i}{\partial V_{FP}} = m \quad \text{Equation 4}$$

$$\frac{\partial V_i}{\partial m} = V_{FP} \quad \text{Equation 5}$$

$$\frac{\partial V_i}{\partial c} = 1 \quad \text{Equation 6}$$

### Test Equipment

Agilent 3458A DMM
NI FP-AI-110 FP module and PC
Voltage reducer network
Dataforth Isolation Amplifier SCM5B31-09
Yokogawa DC voltage/current standard

### Calibration procedure

A stable voltage was applied to the test measurement system using the Yokogawa standard. The reference Digital Multimeter was used to measure  $V_i$ . The Field Point data logging system was used to measure the same voltage.

### Uncertainty Components

#### DMM

DMM Specification	Taken from the manufacturer's product specifications
Readings	Logged using the LabView fuel cell calibration program
Temperature	Uncertainty due to temperature variation taken from manufacturer's specifications, $\Delta T$ read from the DMM at time of calibration
Stability of source	Tested independently and found to be insignificant

#### Field Point

FP Specification	Taken from the manufacturer's product specifications
Readings	Logged using the LabView fuel cell calibration program

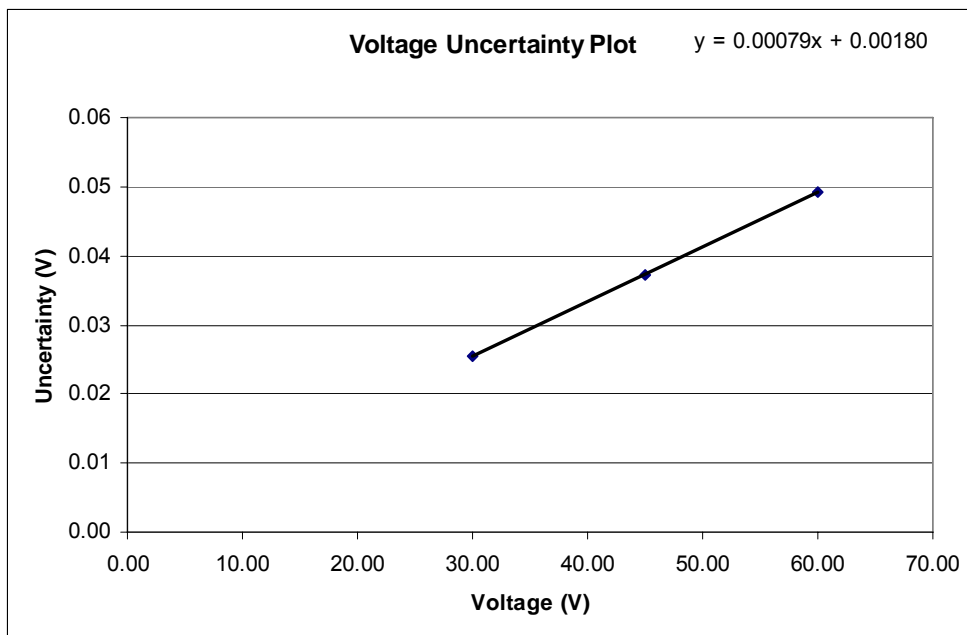
Resolution	Taken from the manufacturer's product specifications
Temperature	Maximum variation due to temperature taken from manufacturer's specifications, assumed to be $\pm 15$ degC.
Common mode signal	Tested independently and found to be less than the resolution
Repeatability	
Drift	
Scaling factor error	Derived from the logged FP data from a linear regression analysis
Offset error	Derived from the logged FP data from a linear regression analysis
Stability of voltage reducer network	The errors are accounted for in the FP system calibration.
Stability of 5B module	The errors are accounted for in the FP system calibration.

### Uncertainty calculations

Calculations have been performed according to Cook R. R. (2002) "Assessment of Uncertainties for Measurement for Calibration and Testing Laboratories", Second Edition, National Association of Testing Authorities, Australia.

All uncertainties derived from logged data have been calculated using a type A analysis. This is a statistical evaluation that assumes the errors to be spread according to a normal distribution curve. All other uncertainties, such as manufacturer specifications, have been calculated using a type B analysis. For this type of evaluation it has been assumed that the errors are spread according to a rectangular distribution, which is more conservative than assuming a normal distribution.

Uncertainty calculations were performed at three points within the testing range. A linear calibration uncertainty (if applicable) was then calculated for the range. This was then used in the measurement uncertainty calculations. All uncertainties were calculated using a nominal coverage factor of 2, which gives an approximate confidence level of 95%.



### Calibration Uncertainty for Voltage

Test Point Input	60	Volts				
Agilent 3458A DMM Specification (Specified Accuracy) for Input DC Voltage						
Range(V)	PPM of Reading	PPM of Range	T_Coeff PPM Reading	T_Coeff PPM Range	Output DMM Specs Accuracy(V)	Resolution of DMM
100	6	0.300	0.15	0.10	0.00039	1.00E-06

T_error	1
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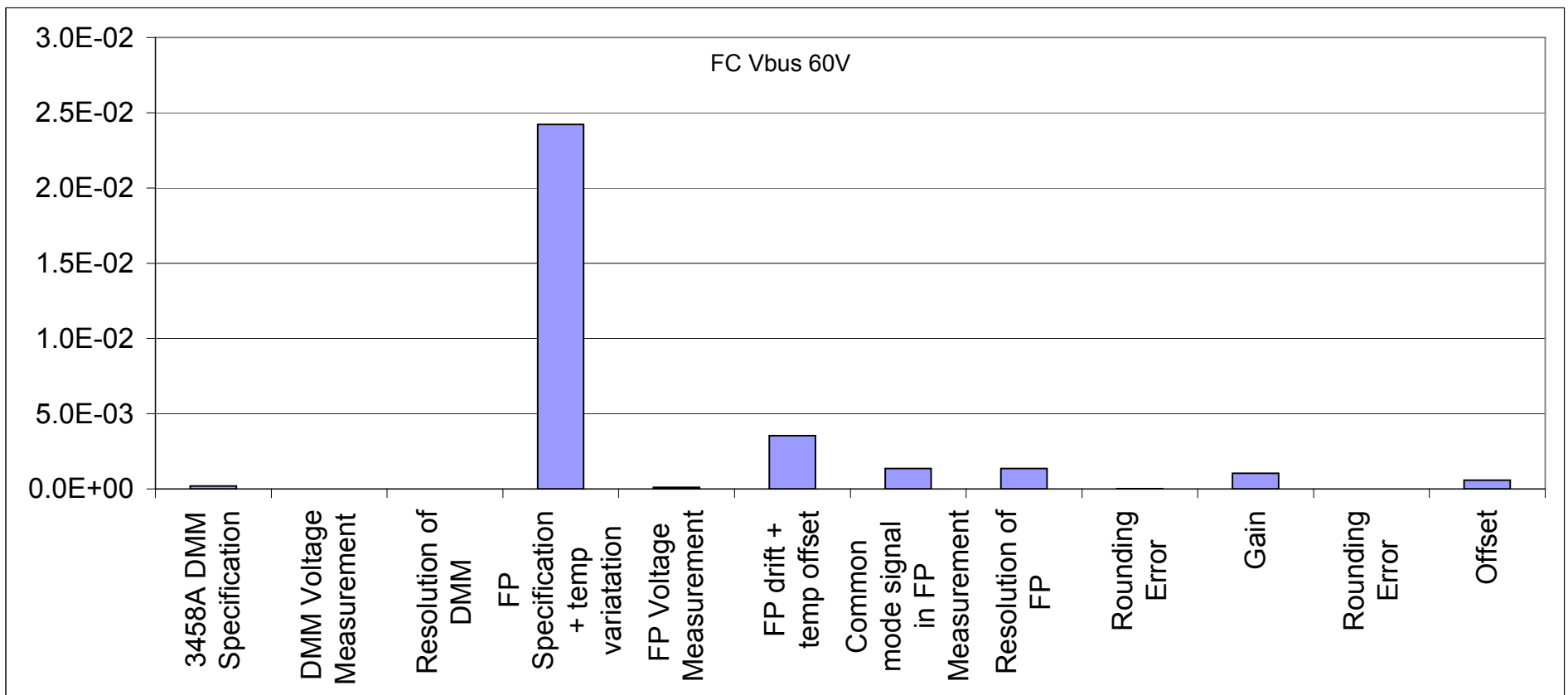
Field Point Specification FP-AI-110					
Range	Range	%reading	Offset	Noise	Resolution
-5.0052V to	10.0104	0.0700%	4.95E-04	1.90E-04	1.90E-04

Datapoint statistics			
	Readings	SD	n
DMM	59.964414	0.00002	91
Vfp	4.833532143	7.50893E-05	91
c	0.009515327	0.000583665	12
m	12.40356589	0.000215819	12

Sensitivity coefficients	
C1	1
C2	12.40356589
C3	4.83353214
C4	1

Item	Description	Uncertainty Component	Units	Distribution Type	Evaluation Type	Range U or a or s	Divisor or d or √n	Degrees of Freedom	Std Uncertainty (u)	Sensitivity coefficients	c <sub>i</sub> u <sub>i</sub>	(c <sub>i</sub> u <sub>i</sub> ) <sup>2</sup>	(c <sub>i</sub> u <sub>i</sub> ) <sup>4</sup> /v	
Vin	3458A DMM Specification	Voltage	V	Normal	B	3.898E-04	2.000	60	1.95E-04	1	1.95E-04	3.80E-08	2.40E-17	C1
Vin	DMM Voltage Measurement	Voltage	V	Normal	A	2.000E-05	9.539	90	2.10E-06	1	2.10E-06	4.40E-12	2.15E-25	C1
Vin	Resolution of DMM	Voltage	V	Rectangular	B	5.000E-07	1.732	60	2.89E-07	1	2.89E-07	8.33E-14	1.16E-28	C1
Vfp	FP Specification + temp variation	Voltage	V	Rectangular	B	3.383E-03	1.732	60	1.95E-03	12	2.42E-02	5.87E-04	5.75E-09	C2
Vfp	FP Voltage Measurement	Voltage	V	Normal	A	7.509E-05	9.539	90	7.87E-06	12	9.76E-05	9.53E-09	1.01E-18	C2
Vfp	FP drift + temp offset	Voltage	V	Rectangular	B	4.950E-04	1.732	60	2.86E-04	12	3.54E-03	1.26E-05	2.63E-12	C2
Vfp	Common mode signal in FP Measurement	Voltage	V	Rectangular	B	1.900E-04	1.732	60	1.10E-04	12	1.36E-03	1.85E-06	5.71E-14	C2
Vfp	Resolution of FP	Voltage	V	Rectangular	B	1.900E-04	1.732	60	1.10E-04	12	1.36E-03	1.85E-06	5.71E-14	C2
m	Rounding Error	V/V	V/V	Rectangular	B	5.000E-06	1.732	60	2.89E-06	5	1.40E-05	1.95E-10	6.32E-22	C3
m	Gain	V/V	V/V	Normal	A	2.158E-04	1.000	11	2.16E-04	5	1.04E-03	1.09E-06	1.08E-13	C3
c	Rounding Error	Voltage	V	Rectangular	B	5.000E-06	1.732	60	2.89E-06	1	2.89E-06	8.33E-12	1.16E-24	C4
c	Offset	Voltage	V	Normal	A	5.837E-04	1.000	11	5.84E-04	1	5.84E-04	3.41E-07	1.06E-14	C4

SUMS	3.47E-03	6.05E-04	5.75E-09
Combined Uncertainty Volts	0.024593885		
Effective Degrees of Freedom	64		
Coverage Factor	2.00		
Expanded Uncertainty Volts	0.04915		
	0.082%		



# Measurement Uncertainties for the Fuel Cell Testing Facility

## Final uncertainty calculation for the Total Energy Input ( $Ql_{total}$ )

### Model Equations

$$Ql_{total} = M_f R_1 \times t \times LHV_{h2} + M_o R_1 \times t \times C_{op} \times T_{of} \quad \text{Equation 1}$$

### Sensitivity determination

$$\frac{\partial Ql_{total}}{\partial M_f R_1} = t \times LHV_{h2} \quad \text{Equation 2}$$

$$\frac{\partial Ql_{total}}{\partial LHV_{h2}} = M_f R_1 \times t \quad \text{Equation 3}$$

$$\frac{\partial Ql_{total}}{\partial M_o R_1} = t \times C_{op} \times T_{of} \quad \text{Equation 4}$$

$$\frac{\partial Ql_{total}}{\partial C_{op}} = M_o R_1 \times t \times T_{of} \quad \text{Equation 5}$$

$$\frac{\partial Ql_{total}}{\partial T_{of}} = M_o R_1 \times t \times C_{op} \quad \text{Equation 6}$$

### Uncertainty Components

<b>MfR_1</b>	Measurement uncertainty of the hydrogen MFM, from calculations
<b>Time (t)</b>	The uncertainty of the logging period has been assumed to be negligible and is therefore ignored.
<b>LHVh2</b>	Hydrogen specification is High Purity (99.99% H2). Therefore the uncertainty of the Lower Heating Value has been assumed to be not greater than 0.01%.
<b>MoR_1</b>	Measurement uncertainty of the air MFM, from calculations
<b>Cop</b>	The uncertainty related to the specific heat of air constant. This has been assumed to be no greater than 2% of the value.
<b>Tof</b>	Measurement uncertainty from temperature calculations

### Uncertainty Calculations

Calculations have been performed according to Cook R. R. (2002) "Assessment of Uncertainties for Measurement for Calibration and Testing Laboratories", Second Edition, National Association of Testing Authorities, Australia.

All uncertainties derived from logged data have been calculated using a type A analysis. This is a statistical evaluation that assumes the errors to be spread according to a normal distribution curve. All other uncertainties, such as uncertainties calculated from manufacturer specifications, have been calculated using a type B analysis. For this type of evaluation it has been assumed that the errors are spread according to a normal distribution. This type of

distribution was chosen as the uncertainties had already been normalised in the calibration uncertainty calculations.

Where appropriate, the uncertainty calculations were performed at three points within the testing range. A linear measurement uncertainty (if applicable) was then calculated for the range. This was then presented as the final measurement uncertainty. All uncertainties were calculated using a nominal coverage factor of 2, which gives an approximate confidence level of 95%.

**Table of Calibration Uncertainties**

Field	Range	Percentage of Measurement (%)	Additional Uncertainty ( $\pm$ )	Units
Current	0 to 150 Amps	0.09%	0.05	A
Voltage	0 to 60 VDC	0.08%	0.002	V
H2 Mass Flow	full range	2.01%	N/A	kg/s
Air Mass Flow	full range	3.10%	N/A	kg/s
Temperature	0 to 80 degC	N/A	0.36	degC
Volumetric Flow	full range	0.50%	N/A	L/s
Pressure	full range	0.91%	N/A	kPag

**Table of Measurement Uncertainties**

Field	Range	Percentage of Measurement (%)	Additional Uncertainty ( $\pm$ )	Units
Current	0 to 15 Amps	0.15%	0.05	A
Voltage	50 VDC	N/A	0.04	V
H2 Mass Flow	0 to 1.1E-4 kg/s	2.02%	N/A	kg/s
Air Mass Flow	0 to 8.0E-3 kg/s	3.11%	N/A	kg/s
Temperature	60 degC	N/A	0.37	degC
Volumetric Flow	0 to 0.4 L/s	0.50%	N/A	L/s
Pressure	445 kPag	0.91%	N/A	kPag
Electrical Power	0 to 1000 W	0.15%	2.60	W
Heat Output	0 to 5000 W	2.10%	N/A	W
Total Energy Input	0 to 12.5 kW	2.30%	N/A	W
Thermal Efficiency	0 to 50 %	3.05%	N/A	%
Electrical Efficiency	0 to 50 %	2.31%	N/A	%

### Measurement Uncertainty for Total Energy Input ( $Q_{I, total}$ )

Test Point Input **750 kJ**  $Q_{I, total} = Q_f + TQ_o = MfR\_1 * LHVh2 + MoR\_1 * Cop * Tof$

Fuel Cell Testing Facility Measurement Uncertainties					
Field	Measured Value	Percentage of Measurement (%)	Additional ( $\pm$ )	Units	Output FCTF Accuracy
MfR_1	1.91799E-05	2.020%	0.000	kg/s	3.87435E-07
LHVh2	120000	0.010%	0.000	kJ/kg	1.20000E+01
Tof	302.1053	0.000%	0.370	K	3.70000E-01
Cop	1.0049	2.000%	0.000	kJ/(kg.K)	2.00980E-02
MoR_1	0.0024	3.110%	0.000	kg/s	7.418E-05

Sensitivity Coefficients		$Q_{I, total}$ (kJ)
C1	7200000.00	MfR_1
C2	0.00	LHVh2
C3	0.14	Tof
C4	43.23	Cop
C5	18215.14	MoR_1

Item	Description	Uncertainty Component	Units	Distribution Type	Evaluation Type	Range U or a or s	Divisor k or d or $\sqrt{n}$	Degrees of Freedom	Std Uncertainty ( $u$ )	Sensitivity coefficients	$c_i u_i$	$(c_i u_i)^2$	$(c_i u_i)^4 / v$	
MfR_1	Hydrogen Mass Flow Rate	mass flow	kg/s	Normal	B	3.874E-07	2.000	60	1.94E-07	7200000.00	1.39E+00	1.95E+00	6.31E-02	C1
LHVh2	Lower Heating Value of Hydrogen	heat value	kJ/kg	Normal	B	1.200E+01	2.000	60	6.00E+00	0.00	6.90E-03	4.77E-05	3.79E-11	C2
Tof	Temperature of Air	temperature	K	Normal	B	3.700E-01	2.000	60	1.85E-01	0.14	2.66E-02	7.08E-04	8.35E-09	C3
Cop	Specific Heat of Air	specific heat	kJ/(kg.K)	Normal	B	2.010E-02	2.000	60	1.00E-02	43.23	4.34E-01	1.89E-01	5.94E-04	C4
MoR_1	Air Mass Flow Rate	mass flow	kg/s	Normal	B	7.418E-05	2.000	60	3.71E-05	18215.14	6.76E-01	4.56E-01	3.47E-03	C5
SUMS									6.20E+00			2.59E+00	6.71E-02	
Combined Uncertainty												1.609754936		
Effective Degrees of Freedom													100	
Coverage Factor													1.98	
Expanded Uncertainty													3.19371	
													2.28%	

